NRF-OeAD Workshop WTZ Korea-Austria

"Surface generation via integrable evolution of curves"

Tuesday, February 3, 2026: Colloquium Day

Program

09:15–09:55 **Joseph Cho**

Monodromy of polarised curves under Darboux transformations.

09:55–10:15 **Sara Bentrifa**

Analytic continuation of rotational linear Weingarten surfaces in hyperbolic space.

Coffee Break

10:45–11:05 **Daren Thimm**

Irregular Factorizations Of Quadratic Conformal Motion Polynomials.

11:05–11:45 **Hwan Pyo Moon**

Approximation methods for tensor product Gauss–Legendre patches.

Lunch Break

13:30–14:10 **Hans-Peter Schröcker**

The Construction of Bistable Four-Bars, Quad Nets, and Cubes.

14:10–14:30 **Andreas Mair**

Curves on the Study Quadric and the Factorization of Closed-loop Linkages.

Coffee Break

15:00–15:20 **Wonjoo Lee**

Bernstein-type theorem for constant mean curvature surfaces in the three-dimensional isotropic space.

15:20–16:00 **Seong-Deog Yang**

Extensions of the Björling Problem.

Conference Dinner at 18:00

Location

University of Innsbruck

Technikerstrasse 13, 6020 Innsbruck

Seminar room 414, 4th floor

Conference Dinner

Isserwirt Lans
Dorfstrasse 9, 6072 Lans
Public Transport: Bus stop *Lans Sennerei* (Bus M)

Participants

Project Members

Joseph Cho (Handong Global University), *organizer*

Udo Hertrich-Jeromin (TU Wien)

Wonjoo Lee (Korea University)

Martin Pfurner (University of Innsbruck)

Gudrun Szewieczek (University of Innsbruck), *organizer*

Seong-Deog Yang (Korea University)

External Speakers

Sara Bentrifa (TU Wien)

Andreas Mair (University of Innsbruck)

Hwan Pyo Moon (Dongguk University)

Hans-Peter Schröcker (University of Innsbruck)

Daren Thimm (University of Innsbruck)

Abstracts

Analytic continuation of rotational linear Weingarten surfaces in hyperbolic space.

Sara Bentrifa (TU Wien)

We are interested in the analytic continuation of linear Weingarten surfaces across the ideal boundary of hyperbolic space. The main focus of the talk is on rotational surfaces with constant Gauss curvature, where we show that the analytic continuation is given by the 180-degree rotation of the profile curve around the boundary curve.

Monodromy of polarised curves under Darboux transformations.

Joseph Cho (Handong Global University)

The study of surfaces largely concerns either local or global aspects, two perspectives that are closely intertwined: local characterizations provide surface generation methods crucial to finding examples with global constraints. A prototypical instance is the Weierstrass representation of minimal surfaces, which facilitates the construction of globally significant examples such as Costa-Hoffman-Meeks surfaces.

Isothermic surfaces have recently seen a revival of interest, and their local theory has been developed from various viewpoints, with one result being the transformations of isothermic surfaces that allows for the generation of new isothermic surfaces from a given one. The focus is now shifting toward the global theory of isothermic surfaces. In this talk, we focus on the theory of polarized curves that serve as a simplification for considering the global properties of isothermic surfaces under transformation theory, and investigate their monodromy problems.

Bernstein-type theorem for constant mean curvature surfaces in the three-dimensional isotropic space

Wonjoo Lee (Korea University)

The classical Bernstein theorem states that any entire minimal graph in the three-dimensional Euclidean space must be a plane. By extending and modifying this theorem, Bernstein-type theorems have appeared in various forms, depending on whether the mean curvature H is zero or not and what the ambient space is.

In this talk, we present a value distribution theorem related with Gaussian curvature of complete spacelike constant mean curvature (CMC) surfaces in the three-dimensional isotropic space $\mathbb{I}^3$, which implies Bernstein-type theorem for CMC-H graphs in $\mathbb{I}^3$. In the following, we give some specific examples related to the results.

This talk is based on the joint work with Shintaro Akamine and Seong-Deog Yang.

Curves on the Study Quadric and the Factorization of Closed-loop Linkages.
Andreas Mair (University of Innsbruck)

This talk provides an introduction to the Study quadric $\mathcal{S}_6$, which is the projective embedding of the Lie group SE(3) of special Euclidean displacements, and its connection to overconstrained closed-loop linkages constructed from rotational joints. We highlight the factorization theory for rational curves on this quadric and sketch a possible extension to curves of higher genus. Ongoing research focuses on a purely algebraic-geometric factorization using divisors on curves. Furthermore, we study the intersection of linear subspaces of $\mathbb{P}^7$ with $\mathcal{S}_6$ to investigate their relationship to specific closed-loop linkages.

Approximation methods for tensor product Gauss–Legendre patches.
Hwan Pyo Moon (Dongguk University)

Tensor product Bézier patches are the most fundamental surface primitives used in Computer Aided Geometric Design (CAGD). Although Bézier patches can be defined regardless of degree, it is rare to use a single high degree Bézier patch to design a surface with complicated shape. For high degree polynomial surfaces, it is difficult to manipulate the shape of surfaces using Bézier control nets, because the value of Bernstein basis decreases and the effect of individual control points becomes weaker as the degree increases.

Recently, a new polynomial basis called Gauss–Legendre (GL) polynomials has been developed for the design of high degree curves. We here present some methods of approximating given parametric surfaces using high degree polynomial tensor product patches based on the GL basis. We first determine the boundary curves of the patch and then compute the interior control points of the GL patch using the data such as the tangent vectors and the twist of the reference surface.

The Construction of Bistable Four-Bars, Quad Nets, and Cubes.
Hans-Peter Schröcker (University of Innsbruck)

A four-bar is a mechanical structure composed of four rigid bodies ("links") that are arranged in cyclic order such that the relative mobility between any two consecutive bodies is restricted to be a rotation around a fixed axis ("revolute joint").

Four-bars are generically rigid but there exist examples that exhibit an infinitesimal or even finite flexibility. In our presentation, we are concerned with a family of four-bars that is rigid but, quite exceptionally, allows for two incongruent realizations. Structures of this type are called "bistable" or "snapping".

We present methods to construct snapping four-bars and we relate them to quadrilaterals in a high-dimensional quadric model. In this way, existence of bistable quad nets becomes evident. Yet, it is challenging to actually produce examples via 3D printing. Challenges include, among others, the control of the revolute joint positions as well as their snapping angles to allow for a non-destructive and collision-free snap.

We address these issues by means of discrete differential geometry. Via Whiteley's Averaging Theorem any infinitesimally flexible discrete quad surface (known examples are discrete minimal surfaces) gives rise to a family of snapping quad nets. We demonstrate the effectiveness of this approach by an example of a beautifully designed "snapping cube".

Irregular Factorizations Of Quadratic Conformal Motion Polynomials.
Daren Thimm (University of Innsbruck)

In this talk we study the factorizability of motion polynomials in conformal kinematics.

For this we first give an overview of the mathematical structures used for the analysis, namely Clifford algebras. Next we review the current state of research in factorization theory of conformal motions and develop the notion of irregular factorizability.

We then present how to explicitly construct examples of quadratic motions presenting this behavior and show which types of motions can exist.

Finally we prove uniqueness statements for two notable quadratic motions in special subsets of the space of conformal motions. The circular translation for the subspace of Euclidean motions and the Villarceau motion for the subset of motions with commuting factors.

Extensions of the Björling Problem.

Seong-Deog Yang (Korea University)

The theory of minimal surfaces in three-dimensional Euclidean space $\mathbb{E}^3$ admits natural extensions to other geometries. A central example is the classical Björling problem: given a curve γ and a unit normal vector field N along γ, can one construct a minimal surface containing γ whose normal coincides with N?

Analogous Björling-type problems arise in diverse settings, including:

- minimal surfaces in Euclidean three-space,
- maximal surfaces in Lorentzian three-space,
- zero mean curvature surfaces in isotropic three-space and in the light cone,
- constant mean curvature (CMC) 1 surfaces in hyperbolic and de Sitter three-space.

In this talk, I will survey recent developments on these extensions, emphasizing new representation formulas and geometric insights.